Bayesian Modeling of Extreme Precipitation in Mindanao, Philippines

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Abstract

The Philippines is one of the countries that is prone to typhoons and heavy rainfall. Various atmospheric measures are monitored to inform its citizens regarding climate and weather events. One of these is the r-year return level. In this study, the researchers developed precipitation return level maps with uncertainty measures for selected provinces in Mindanao, Philippines. Using advanced statistical and computational techniques, results demonstrated that a Generalized Pareto Distribution-based Bayesian hierarchical model can effectively estimate r-year precipitation return levels and their associated uncertainty. The hierarchical models efficiently handled the uncertainties in the estimation and easily integrated key covariates in the modeling. It is recommended that more parameters and other covariates be considered to extend the complexity of the model.

Keywords: Bayesian hierarchical model, daily precipitation, generalized pareto distribution, r-year return level, spatial analysis

1. Introduction

Despite the Philippines' well-known vulnerability to storms, Typhoon Washi (locally known as 'Sendong') caught many off-guard when it struck Mindanao at midnight on December 16, 2011. This devastating typhoon resulted in over a thousand fatalities and caused nearly a billion pesos in damages. An estimated 18 inches of rain fell in Cagayan de Oro alone, more than half the city's average monthly total. Such extreme precipitation events underscore the importance of understanding their occurrence for effective disaster risk management.

A common measure of extreme weather events is the return level. The return level for a given return period T (e.g., 10 years) is the rainfall amount expected to be exceeded on average once every T year. For instance, if the 10-year return level for daily rainfall in a particular region is 10 inches, a daily rainfall event of 10 inches or more is expected to occur, on average, once every 10 years.

Regional Frequency Analysis (RFA), introduced by Dalrymple (1960), has been widely applied in hydrology and engineering for estimating return levels over several decades (Hosking and Wallis, 1997). However, RFA has notable limitations, including its inability to incorporate covariates into parameters and its restricted capacity to evaluate error propagation through its multi-step process (Katz *et al.*, 2002). Bayesian hierarchical modeling (BHM) has emerged as a powerful alternative for analyzing spatial extremes (Cooley *et al.*, 2007). This approach enables simultaneous inference of all unknown quantities, effectively accounting for interactions between estimation errors across different levels.

This study applied BHM to estimate five-, three-, and five-year return levels for daily precipitation in Mindanao, Philippines. BHM's predictive performance was validated, and localized return level estimates were obtained, addressing the island's unique challenges with extreme precipitation.

2. Methodology

2.1 Extreme Value Theory (EVT)

Given independent and identically distributed random variables $Z_1, Z_{2,...,}$ and letting $B_n = max(Z_1, Z_{2,} ..., Z_n)$ represent the maximum over a block of n values. Under certain regularity conditions, the distribution function of B_n converges to a specific three-parameter distribution, known as the Generalized Extreme Value distribution (Fisher and Tippet, 1928). For a sufficiently large threshold u, the distribution function of the excesses, $Y = Z - u$, conditional on $Z > u$, are described approximately by the Generalized Pareto Distribution (GPD), as shown in equation 1 (Pickands,1975).

$$P\left(Z>z+u \mid Z>u\right) = \left(1+\xi\frac{z}{\sigma_u}\right)_+^{-\frac{1}{\xi}} \tag{1}$$

where $a_+ = a$ if $a \geq 0$ and $a_+ = 0$ if $a < 0$.

The scale parameter σ_u, is greater than zero, and the shape parameter ξ, controls whether the tail is bounded $(\xi < 0)$, light $(\xi \to 0)$or heavy $(\xi > 0)$. In practice, a threshold is chosen at a level where the data above it approximately follows a GPD and the shape and scale parameters are estimated.

EVT provides the link between data recorded on a smaller scale (e.g., daily, hourly) time frame and quantities of longer time scales such as return levels. Letting n_y represent the number of observations taken in a year, one obtains the r-year return level, z_r**,** by solving the equation $P(Z > z_r) = 1/rn_y$ *for* z_r as shown in Equation 2.

$$z_r = u + \frac{\sigma_u}{\xi}\left[\left(rn_y\varsigma_u\right)^{\xi} - 1\right] \quad \text{with } \varsigma_u = P(Z > u) \tag{2}$$

2.2 Data Collection and Study Area

The National Oceanic and Atmospheric Administration website obtained daily precipitation totals (in inches) from January 1, 2010, to December 31, 2023 for 14 weather stations located in the different provinces of Mindanao. For an area like Mindanao with diverse geographies but with sparse weather stations, elevation and geographical coordinates will likely influence on the climatological behavior of extreme precipitation. Thus, the corresponding latitude, longitude, and elevation were obtained as covariates in the modeling. Figure 1 shows the Map of the study area (in yellow shade), which includes provinces from Region IX (Zamboanga Peninsula), Region X (Northern Mindanao), and Region XI (Davao Region).

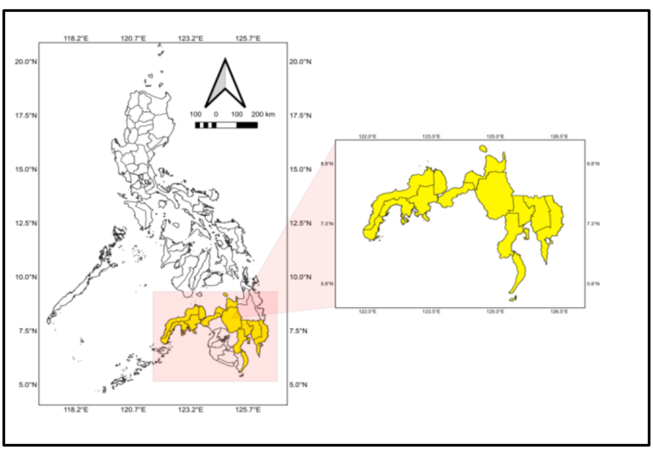

Figure 1. The Map of the study area

2.3 Threshold Selection

Threshold selection is crucial in fitting a BHM based on the GPD. This study utilized Wadsworth test diagnostics, which returns the white noise process and calculates the null Distribution. This selects the lowest threshold value at which the hypothesis of the change point is not rejected for all higher thresholds (Wadsworth, 2012).

2.4 Data Declustering

After determining the optimal threshold value, precipitation values for each province were declustered. Only precipitation values exceeding the optimal threshold were used to fit the GPD-based BHM.

2.5 Maximum Likelihood Estimation of the Scale Parameter

An algorithm from Cooley (2007) was applied to conduct maximum likelihood estimation of the scale parameter for the declustered data. This estimates the scale parameter for each location since one of the model's assumptions is that the scale parameter of each location considered is known.

2.6 The Bayesian Hierarchical Model

In this study, four BHMs were formulated to estimate return levels. The formulation was discussed in three layers. It was based on the method proposed by Cooley (2007) and O'Sullivan (2019).

2.6.1 The Data Layer

Suppose we have s locations denoted by $x_1, x_2, \dots, x_s$. Let $Z_k(x_i)$ be the k-th recorded precipitation value at location x_i, modeled as a GPD as shown in Equation 3.

$$z_k(x_i) \sim GPD\left(\mu(x_i), \sigma(x_i), \xi(x_i)\right) \tag{3}$$

where $\mu(x_i)$, $\sigma(x_i)$, *and* $\xi(x_i)$ are the optimal threshold (also known as location parameter), scale, and shape parameters of each location, respectively. The probability that the precipitation value would exceed the threshold was assumed to follow a binomial distribution.

2.6.2 The Process Layer

In this layer, it was assumed that the parameters of the GPD vary smoothly in the latitude-longitude space. It describes the relationship between the latent spatial process, the mean logarithmic transformation of the scale parameters denoted by $\phi(x)$, as a linear relationship of its covariates. In this study, two possible covariates were considered. For the first covariate, the elevation of each location was utilized as a predictor of $\phi(x)$ as depicted in equation 4.

$$E(\phi(x)) = \mu_{\phi}(x) = \alpha_{\phi,1} e_{\phi(x)} + \alpha_{\phi,0} \tag{4}$$

where $e_{\phi(x)}$ is the elevation (in meters). Moreover, latitude and longitude were also considered covariates of the Bayesian model. It can be modeled using a multiple linear relationship concerning the logarithmic transformation of the scale parameters, as depicted in Equation 5.

$$E(\phi(x)) = \mu_{\phi}(x) = \alpha_{\phi,1} lat_{\phi(x)} + \alpha_{\phi,2} long_{\phi(x)} + \alpha_{\phi,0} \tag{5}$$

Aside from that, the logarithmic transformation of the scale parameters is assumed to be from a normal distribution with a uniform precision value denoted by σ_{ϕ}. For the shape parameter, two possible formulations were created. First, it was modeled as a single value with a uniform prior. Another formulation is that the shape parameter values still have a uniform prior; however, it varies in each location.

2.6.3 The Prior Layer

The prior distributions of the parameters indicated at 2.6.2. The prior distribution of regression parameters is assumed normal. The priors of the precision value of the regression coefficients and shape parameters are assumed to be uniform distributions.

2.7 Return Level Estimation

Determining the posterior of the marginal distributions of each location was done using Equation 2. The associated return levels were visualized using geographical choropleth maps, which represent the return level distribution plots for selected provinces in Mindanao. In this study, the only focus was the posterior means and the bounds of the 95% high-density index. The high-density index was defined as the set of all values that fall between the 2.5th and 97.5th percentile, representing the range within which the true parameter value is most likely to lie.

2.8 Markov Chain Monte Carlo (MCMC) Approximation Algorithm

The MCMC approximation was set into four chains in 20,000 iterates with 1,000 burn-in steps. A thinning value of 1 was used to reduce the autocorrelation of the lags. In addition, stationarity tests were utilized to determine whether the chains converged to their stationary distribution. This was done using MCMC diagnostics, specifically the Effective Sample Size (ESS), R - hat, and the Monte Carlo Squared Error (MCSE). This can be done using the NUTS sampler.

2.9 Model Comparison

The performance of the models was compared using the Deviance Information Criterion (DIC). The DIC is a generalization of the Akaike Information Criterion (AIC). It estimates the adequate number of parameters by calculating the difference between the posterior mean of the deviance and the deviance at the posterior means of the parameters. A lower DIC value indicates a better model fit, balancing model complexity and goodness of fit.

2.10 Model Validation

Model validation was conducted by comparing the actual and estimated return level values of the best-fit model, specifically for two, three, and five-year return levels.

3. Results and Discussion

3.1 The threshold and Declustered Data

Based on the Wadsworth diagnostic test (Wadsworth, 2014), the optimal threshold value was determined to be 8.4 inches. Figure 2 shows the time series plot of the data after retaining the precipitation data that exceeded the threshold. Initially, each province had 5,113 observations. Nearly half of the observations from Zamboanga del Norte, Zamboanga del Sur, and Davao de Oro exceeded the threshold. These provinces had higher probabilities of experiencing precipitation values that exceed the threshold, compared with other provinces. When data values at a particular location are more likely to exceed the threshold, the return level increases, indicating a higher frequency of extreme events (O'Sullivan *et al*., 2019).

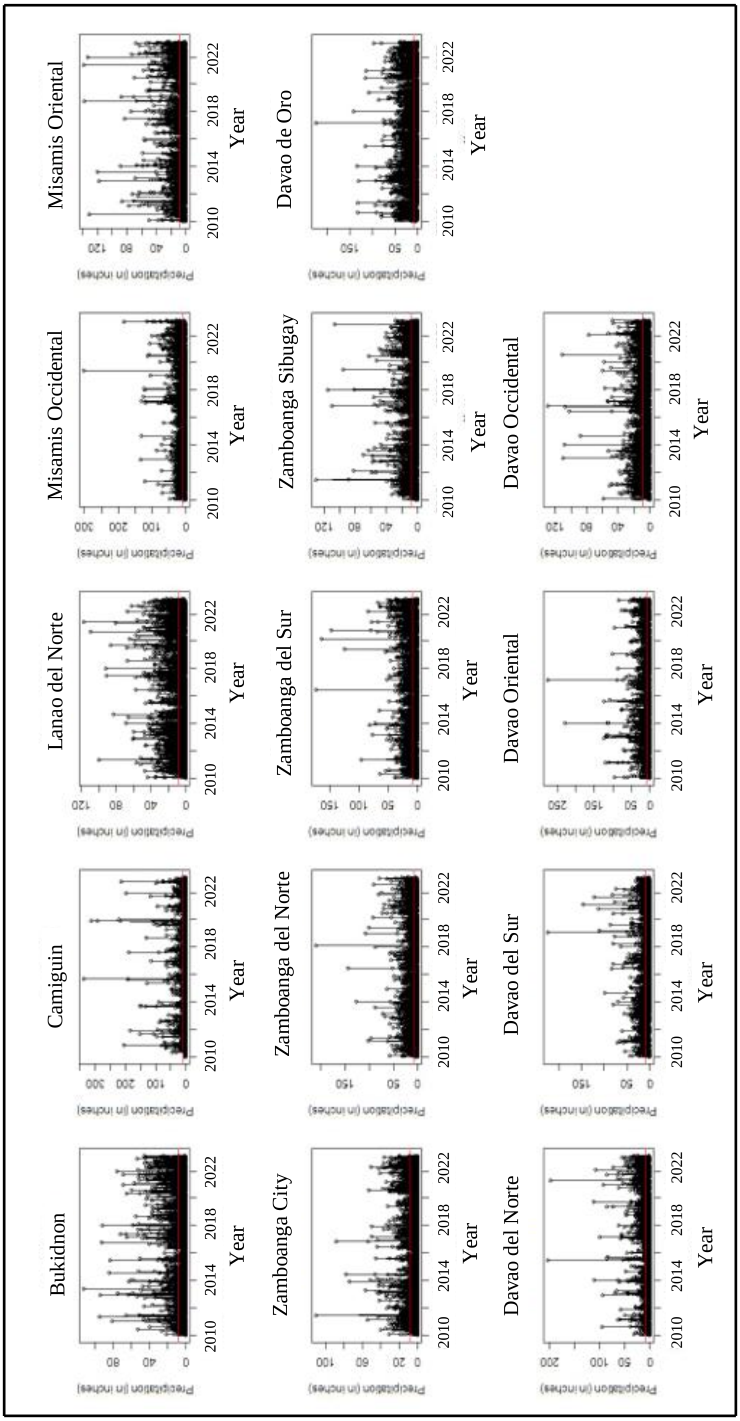

Figure 2. Retained precipitation values after declustering

3.2 Estimates of the Variability of Extreme Values

Figure 3 shows the maximum likelihood estimates of the scale parameter of the GPD for the selected provinces. Davao de Oro had the highest value, while Zamboanga City had the lowest among all provinces. This implied that the extreme values in Davao de Oro were highly dispersed whereas Zamboanga City exhibited low variability. This suggested that Davao de Oro experienced a wide range of extreme precipitation events while the Zamboanga City region experienced more consistent extreme precipitation. As Gilleland and Katz (2016) pointed out, increasing the scale parameter leads to more conservative risk estimates, predicting higher potential losses and extreme events.

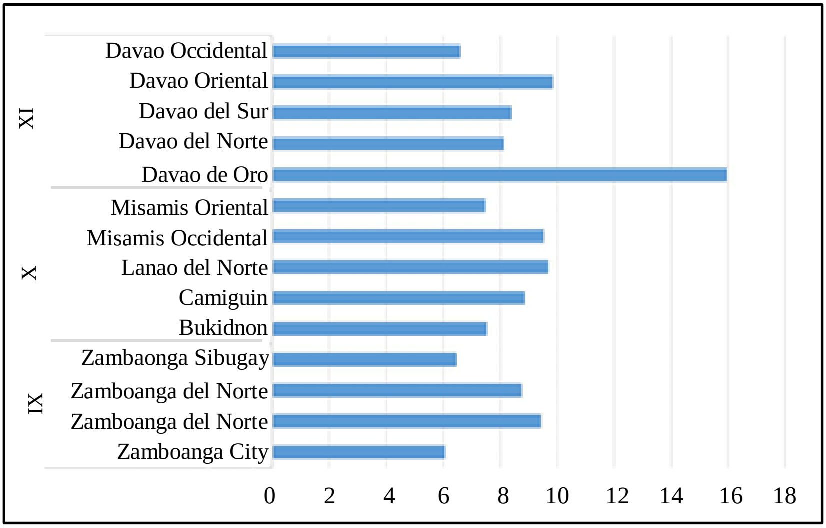

Figure 3 The maximum likelihood estimates of the scale parameter of the GPD

3.3 Posterior Distributions

Tables 1 to 3 summarize the posterior means of the models' parameters and their corresponding credible intervals. The tables that the coefficient distributions for Models 1 and 2 were similar in terms of their posterior means and 95% high-density intervals. The corresponding posterior intercept mean was 2.08, and the posterior slope was 0.00015. Credible intervals were around [1.8, 2.3] and [-0.00025, 0.00055], respectively.

Table 1. Posterior distribution of the GPD parameters (Model 1)

Model	Parameter	Posterior mean	Posterior standard error	95% High density index	
				2.50%	97.50%
Model 1	$\alpha_{\phi,0}$	2.088	0.00108	1.881	2.294
	$\alpha_{\phi,1}$	0.00015	0.000002	-0.00025	0.00055
	α_{ϕ}	0.272	0.00078	0.179	0.43
	ζ(x)	0.149	0.000319	0.143	0.156
	$\alpha_{\phi,0}$	2.087	0.00069	1.883	2.291
	$\alpha_{\phi,1}$	0.00015	0.000001	-0.00025	0.00054
	α_{ϕ}	0.273	0.00052	0.18	0.43
	ζ(x1)	0.146	0.0071	0.123	0.17
	ζ(x2)	0.164	0.01042	0.131	0.2
	ζ(x3)	0.153	0.0067	0.131	0.177
	ζ(x4)	0.157	0.00643	0.136	0.179
	ζ(x5)	0.146	0.00666	0.125	0.168

Additionally, Models 3 and 4 were similar in terms of their slopes and y-intercepts, with a corresponding posterior intercept mean of around -9.3 and a posterior slope mean of (0.03, 0.09), with credible intervals around [-23.9, 4.7] for the intercept, [-0.024, 0.085] for the slope of the latitude, and [-0.02, 0.20] for the slope of the longitude.

Table 2. Posterior distribution of the GPD parameters (Model 2)

Model	Parameter	Posterior mean	Posterior standard error	95% High density index	
				2.50%	97.50%
Model 2	ζ(x6)	0.135	0.00648	0.114	0.159
	ζ(x7)	0.154	0.00609	0.134	0.176
	ζ(x8)	0.15	0.00605	0.131	0.17
	ζ(x9)	0.138	0.00589	0.119	0.158
	ζ(x10)	0.187	0.00878	0.159	0.217
	ζ(x11)	0.15	0.00751	0.126	0.177
	ζ(x12)	0.15	0.00729	0.127	0.175
	ζ(x13)	0.159	0.00714	0.135	0.184
	ζ(x14)	0.138	0.00606	0.119	0.159

The shape parameters of all models were between 0 and 1. This implied that the tail of the extreme distribution for all provinces was less heavy. Small shape parameter values implied that extreme precipitation events are more probable. This can affect strategies for risk reduction and management, and how to mitigate risks (Gilleland and Katz, 2016).

Table 3. Posterior distribution of the GPD parameters (Model 3 and 4)

Model	Parameter	Posterior mean	Posterior standard error	95% High density index	
				2.50%	97.50%
Model 3	$\alpha_{\phi,0}$	-9.395	0.09695	-23.995	4.714
	$\alpha_{\phi,1}$	0.03	0.00032	-0.024	0.085
	$\alpha_{\phi,2}$	0.091	0.00076	-0.02	0.207
	α_{ϕ}	0.258	0.00083	0.164	0.423
	$\zeta(x)$	0.149	0.00282	0.143	0.156
Model 4	$\alpha_{\phi,0}$	-9.284	0.0665	-23.294	4.779
	$\alpha_{\phi,1}$	0.0298	0.02669	-0.024	0.083
	$\alpha_{\phi,2}$	0.09	0.00052	-0.021	0.201
	α_{ϕ}	0.254	0.00055	0.165	0.409
	$\zeta(x1)$	0.146	0.00787	0.123	0.17
	$\zeta(x2)$	0.164	0.01188	0.131	0.2
	$\zeta(x3)$	0.153	0.00739	0.131	0.178
	$\zeta(x4)$	0.157	0.00686	0.136	0.178
	$\zeta(x5)$	0.146	0.00711	0.125	0.168
	$\zeta(x6)$	0.135	0.00768	0.114	0.158
	$\zeta(x7)$	0.154	0.00684	0.134	0.175
	$\zeta(x8)$	0.15	0.00691	0.131	0.17
	$\zeta(x9)$	0.138	0.00663	0.119	0.158
	$\zeta(x10)$	0.187	0.00977	0.158	0.218
	$\zeta(x11)$	0.15	0.00907	0.125	0.177
	$\zeta(x12)$	0.15	0.00788	0.127	0.175
	$\zeta(x13)$	0.159	0.0087	0.135	0.184
	$\zeta(x14)$	0.138	0.00673	0.119	0.159

3.4 Model Comparison

Table 4 shows the models' DIC and corresponding values. The results indicated that Model 2 was the best-fitted model among all the fitted models, with a DIC value of -23,839.92. This was followed by Model 1, which had a DIC value of -4,903.85. Models 3 and 4 had large DIC values of 3,377.69 and 4,903.85, respectively. The model with varying shape parameters for each location best fitted the data using elevation as a predictor in estimating return levels. This indicated that elevation effectively predicted the return level of each province compared to the geographical coordinates.

Table 4. Model performance measures

Model	D(bar)	P_D	DIC
Model 1	22727.04	-13815.4	-4903.85
Model 2	28021.31	-25930.6	-23839.92

3.5 Model Validation

Tables 5 to 6 show the actual and fitted two, three, and five-year return levels of the best-fit model based on section 3.4.

Table 5. Actual return levels

	Two-year return level estimates			Three-year return level estimates			Five-year return level estimates		
Location	Posterior mean	2.5%	97.5%	Posterior mean	2.5%	97.5%	Posterior mean	2.5%	97.5%
Bukidnon	67.14	63.43	71.56	73.86	69.39	79.24	82.92	77.33	89.70
Camiguin	73.22	67.74	80.07	81.38	74.66	89.87	92.47	83.92	103.40
Lanao del Norte	93.27	87.60	100.03	102.77	95.96	110.94	115.61	107.16	125.84
Misamis Occidental	97.34	91.48	104.15	107.16	100.16	115.35	120.45	111.80	130.67
Misamis Oriental	69.21	65.50	73.56	76.03	71.57	81.31	85.23	79.68	91.86
Zamboanga City	51.27	48.84	54.20	56.20	53.26	59.78	62.82	59.13	67.35
Zamboanga del Norte	95.90	90.41	102.48	105.48	98.93	113.39	118.44	110.33	128.29
Zamboanga del Sur	88.00	83.24	93.65	96.64	90.96	103.42	108.30	101.29	116.74
Zamboanga Sibugay	59.09	56.32	62.31	64.70	61.38	68.60	72.23	68.10	77.11
Davao de Oro	180.29	164.44	199.87	200.52	181.49	224.27	228.29	204.59	258.23
Davao del Norte	71.09	66.85	76.26	78.42	73.29	84.74	88.32	81.89	96.33
Davao del Sur	76.88	72.33	82.29	84.67	79.19	91.23	95.18	88.35	103.44
Davao Oriental	94.47	88.35	101.86	104.31	96.95	113.26	117.64	108.46	128.90
Davao Occidental	59.87	57.04	63.15	65.58	62.18	69.55	73.25	69.02	78.22

The tables demonstrated that the fitted return level values of Model 1, which utilized the elevation of each province as a predictor of the logarithmic transformation of the scale parameter, were close to their corresponding actual return level values. There was a slight difference between the actual and predicted values in terms of their posterior means, which represent the average return levels.

Table 6. Fitted return levels

Location	Two-year return level estimates			Three- year return level estimates			Five-year return level estimates		
	Posterior mean	2.5%	97.5%	Posterior Mean	2.5%	97.5%	Posterior Mean	2.5%	97.5%
Bukidnon	74.15	52.82	106.62	81.68	57.63	118.56	91.82	64.04	134.83
Camiguin	72.16	47.14	114.90	80.19	51.66	129.46	91.09	57.70	149.57
Lanao del Norte	79.64	61.18	105.32	87.61	66.76	116.86	98.39	74.22	132.62
Misamis Occidental	96.47	52.53	186.10	106.19	57.14	206.89	119.36	63.32	235.33
Misamis Oriental	73.94	58.47	94.75	81.30	63.80	105.02	91.22	70.90	118.99
Zamboanga City	65.51	52.21	83.44	72.09	57.00	92.59	80.91	63.36	104.99
Zamboanga del Norte	89.52	58.08	142.27	98.40	63.23	157.80	110.42	70.14	179.01
Zamboanga del Sur	86.36	59.10	129.37	94.82	64.33	143.25	106.24	71.33	162.15
Zamboanga Sibugay	73.16	55.07	98.86	90.33	59.99	109.41	89.96	66.54	123.70
Davao de Oro	104.29	62.81	179.90	115.58	68.75	201.75	131.07	76.80	232.17
Davao del Norte	70.77	55.40	91.97	78.06	60.57	102.41	87.91	67.48	116.67
Davao del Sur	74.48	58.15	96.82	81.99	63.49	107.53	92.13	70.62	122.14
Davao Oriental	79.15	61.46	103.55	87.23	67.17	115.16	98.19	74.81	131.09
Davao Occidental	82.47	45.35	158.49	90.69	49.25	176.03	101.72	54.45	199.80

3.6 The Return Level Maps with Uncertainty Measures

Figures 4 to 6 show the return level maps for the selected provinces in Mindanao. These maps displayed the posterior mean and associated 95% high density interval (HDI) for the two-, three-, and five-year return levels. Provinces in the Davao Region and Zamboanga Peninsula had the highest return level values compared with other regions.

To illustrate, the mean return levels for extreme precipitation events over different time periods in Davao de Oro were as follows:

Two-year return level was 104.3 inches, with a 95% probability that the true return level was between 62.8 and 179.9 inches.

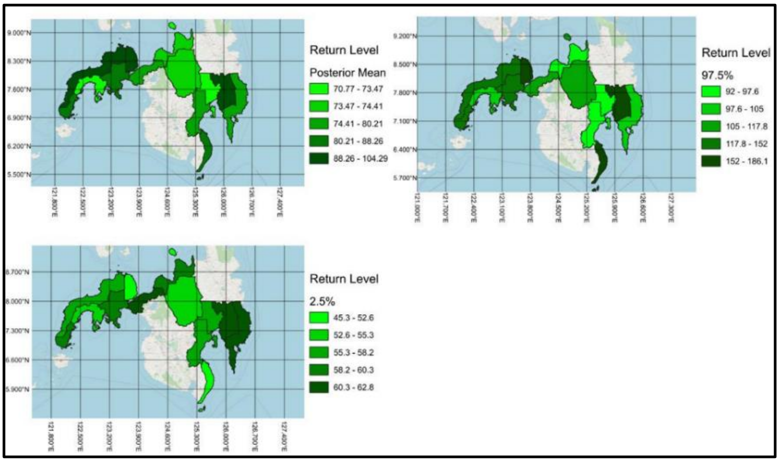

Figure 4. Two-year return level plots of the selected provinces in Mindanao

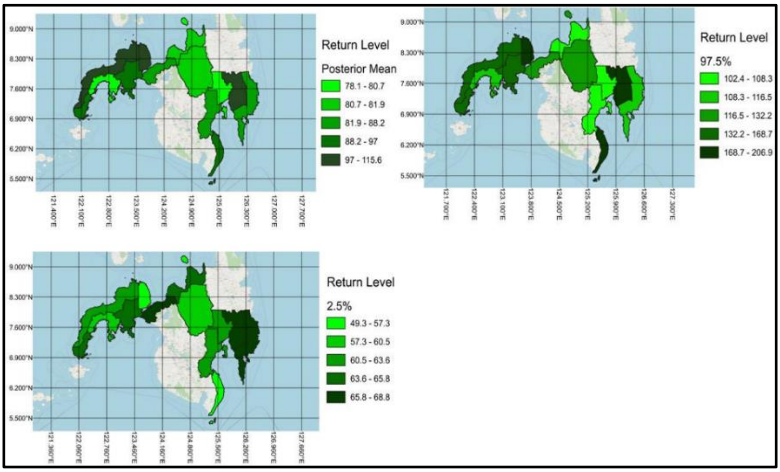

Figure 5. Three-year return level plots of the selected Provinces in Mindanao

Three-year return level was 115.6 inches, with a 95% probability that the true return level was between 68.8 and 201.7 inches. Five-year return level was 131.1 inches, with a 95% probability that the true return level was between 76.8 and 232.2 inches. This means that, on average, an extreme precipitation event with a magnitude of 104.3 inches is expected to occur once every two years, an event with a magnitude of 115.6 inches is expected to occur once every three years, and an event with a magnitude of 131.1 inches is expected to occur once every five years.

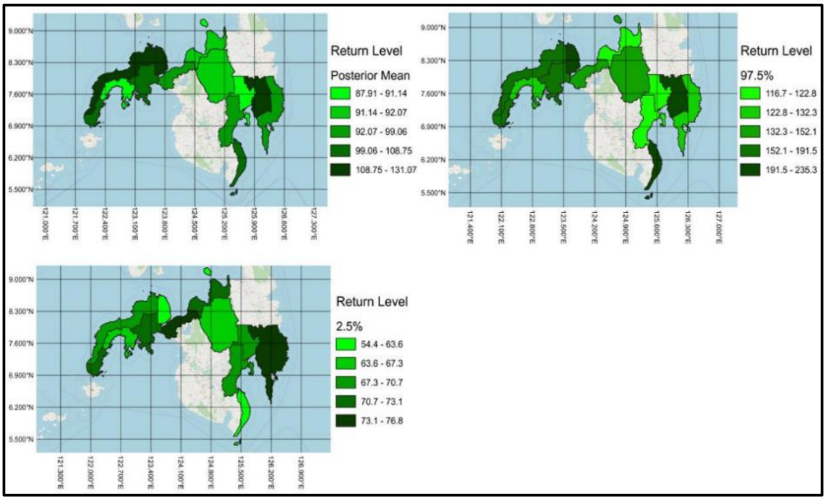

Figure 6. Five-year return level plots of the selected provinces in Mindanao

The credible intervals indicate a 95% probability that the true return levels for these periods will fall within the specified ranges. This provides a measure of uncertainty around the estimates, helping to understand the range of possible extreme values and the confidence in the model's predictions.

4. Conclusion and Recommendation

Bayesian hierarchical models can efficiently estimate selected provinces' daily precipitation return levels using elevation and varying shape parameter values. The shape parameters of the GPD of all provinces implied that the extreme values of all provinces occur most probable. The return level maps in disaster risk management and long-term planning. It is recommended that more parameters and other covariates be considered to extend the complexity of the model.

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