Determination of Discharge Coefficients by Drop Rate Method Using Ultrasonic Level Sensor

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Abstract

The study of cheaper alternatives to commercially available flow meters in the market continues because cost has since been one of the major factors considered in instrument selection. This study presents the results of an experimental investigation involving the use of Unplastized Polyvinyl Chloride (UPVC) pipe male adapter as an alternative variable head flow meter. Seven test specimens of different geometries were made from NELTEX® 20 mm UPVC pipe male adapter having a throat to diameter ratio, β, of 0.665. The results showed that the values of coefficient of discharge corresponding to geometric variation of the adapter can be determined accurately using an open tank draining model as proven by Torricelli's Law. Of the seven specimens tried, the shorter and tapered specimen is the better meter geometry based on its higher values of coefficients of discharge which in effect lead to faster discharge times.

Keywords: discharge coefficients; ultrasonic level meter; variable head meters flow meters

1. Introduction

In recent years there has been a considerable development of sophisticated flow meters. It is estimated that at least 100 flow meter types are commercially available (Aichouni *et al.*, 2000). New types of instruments like ultrasonic meters, vortex meters, and coriolis meters are continually introduced in the market. These instruments usually have the added digital features which make them more expensive than the conventional types like the variable head meters or otherwise known as differential pressure meters.

For some common industry applications, these new types of instruments are not cost effective. The use of more advanced and expensive instruments may not be practical to some simple applications in the industry especially if their added features are not fully utilized or not necessary at all. Applications like vented tank discharge metering and flow rate maintenance may not require sophisticated instrumentation.

This study was conducted to determine the values of coefficient of discharge corresponding to geometric variation of the adapter using an open tank draining model as proven by Torricelli's Law. Furthermore, the identified values of coefficient of discharge were used to calculate flow rates.

Also this study was conducted to investigate a cheaper alternative to commercially available flow meters in the market. This study was specifically an experimental investigation involving an Unplastized Polyvinyl Chloride (UPVC) pipe male adapter tried as a variable head flow meter. A UPVC pipe male adapter is shown in Figure 1.

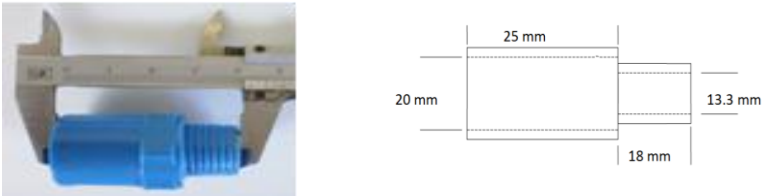


Figure 1. 20 mm UPVC pipe male adapter *(Neltex brand)*

A major significance of using this adapter is its lower cost and availability. Whenever this UPVC pipe male adapter is installed in any vented tank discharge metering applications, this fitting will serve simultaneously as an adapter and flow meter. All measurements can be made, in conjunction with a level meter and Torricelli's equation, to determine discharge flow rates. Consequently both level and flow rates are measured in the process.

Recently, there has been an extensive development of sophisticated flow meters. These developments usually translate to a high price tag. Though it is advantageous to use these new instruments due to their digital capabilities, the problem is they are not cost effective for use in some other applications. Possibly some of their added features may not be needed after all. For this reason there is a need to study cheaper alternatives that could be of more practical use for intended applications without sacrificing efficiency and safety.

The study sought to address the following important issues: (a) insufficiency of cheaper meters intended for measuring fluid discharge in vented tanks; (b) unavailability of coefficient of discharge data for flow meter alternatives like UPVC pipe male adapters; and (c) establish the accuracy and repeatability of this alternative meter. The main objective was to determine the discharge coefficients of water flowing from the side of a vented tank through PVC Pipe adapters of various geometry as follows: (1) long, squared edge; (2) long, tapered edge; (3) long, 45 degrees cut, squared edge; (4) long, 45 degrees cut, tapered edge; (5) long, 30 degrees cut, squared edge; (6) short, squared edge; and (7) short, tapered edge. Moreover, the study aimed to correlate results with available theories and other studies; to determine the influence of adapter length, angle of cut, and edge tapering on coefficient of discharge; to show the relation of coefficient of discharge between tank level, flow rate and Reynolds number; and, to compare numerical results with analytical calculations.

The study focused on the performance of the UPVC pipe male adapter as a flow meter in the vented tank model setup. A single brand (NELTEX) and size of UPVC pipe male adapter was used in this study. The adapter has a 20 mm inlet and 13.3 mm throat dimensions corresponding to a throat to diameter ratio, β, of approximately 0.655.

This study used a vented tank draining model having a uniform cross section only and without surface deformation. A 20 liter standard lubricant container was modified into a tank for conducting basic draining experiment. The tank was designed for incorporation of UPVC pipe adapters and an ultrasonic level sensor measuring device. The tank successfully modeled draining flow as proven by Torricelli's equation. Tank level measurements are limited up to the sensing range of the ultrasonic level sensor.

The fluid considered in this study was only water and its temperature was taken constant at 30 degrees Celsius. Two phase fluid tendencies in turbulent conditions were not investigated due to its complexity. During the study, variations in local acceleration due to gravity and ambient temperature were assumed to be negligible.

2. Methodology

2.1 Theoretical Formulation

2.1.1 Calculation of Coefficient of Discharge

The theoretical discharge, Q_t, of water discharging at the side of the tank through an adapter was calculated by relating the hydrostatic pressure head (liquid level from outlet) to solve the fluid outlet velocity using Torricelli's law. The ideal discharge would be the discharge achieved without friction. The theoretical discharge, Q_t, is the product of outlet velocity and area, or:

$$Q_t = (A_p)\,(v_T) \tag{1}$$

where

$A_p = \pi\,(D_p)^2/4$ is the area of flow outlet, in mm^2;
where D_p is the diameter of outlet pipe, in mm;

and

$v_T = \sqrt{2gh}$ (by Torricelli's equation) is the discharge pipe outlet velocity, in mm/s;
where g is gravitational acceleration (= $9.81\ m/s^2$);
and, h is the tank level reading, in mm.

The actual discharge, Q_a, was measured indirectly using the ultrasonic level sensor meter. The tank draining data logged by the sensor software (Ultra 2001) were used to determine the level drop rates after the instrument error was considered. It is the discharge that occurs and which is affected by friction as the jet passes through the meter. Mathematically, the actual discharge, Q_a, is the product of level drop rate and the tank cross sectional area, or:

$$Q_a = (A_t)\,\Delta L/\,t \tag{2}$$

where

$A_t = \pi\,(D_t)^2/4$ is area of tank, in mm^2;
where D_t is the diameter of tank, in mm;

and,

$(\Delta L/t)$ is the level drop rate, in *mm/s*.

The coefficient of discharge, C_d, is the ratio of the actual discharge, Q_a, and the theoretical discharge, Q_t, or:

$$C_d = Q_a / Q_t \tag{3}$$

The coefficient of discharge is heavily related to the volumetric flow rate of the fluid flow and the cross sectional area of the meter. It is also related to the gravitational constant and the head pressure.

Different values of discharge coefficients were determined relative to variations in pipe geometry and tank level. A plot of the graph of C_d versus tank level readings, its equivalent flow, and Reynolds number, R_e, were made for interpretation and future measurement reference. Reynolds number, R_e, is expressed as:

$$R_e = (v_T)\,(\rho)\,(D_p)/\mu \tag{4}$$

where ρ and μ are density and viscosity of the fluid, respectively.

At 30^0C, the viscosity and density of water are as follows:

$\mu_{water} = 8 \times 10^{-4}$ *Pa.s;* and,
$\rho_{water} = 996\ kg/m^3$.

The relationship of the coefficient of discharge and Reynolds were established using equations (3) and (4). From equation (4), the outlet velocity, v_T, can be expressed as:

$$v_T = (R_e)\,(\mu)\ /(\rho)\,(D_p) \tag{5}$$

Hence, using equations (1) and (5), the relationship between the coefficient of discharge, C_d, and Reynolds number, R_e, can be expressed as:

$$C_d = \frac{Q_a}{A_p\left(\dfrac{R_e\mu}{D\,\rho}\right)} \tag{6}$$

2.1.2 .Calculation of Level Drop Rates

The level drop rate, $\Delta L/t$, is the derivative, dL/dt, of the trend line of level measurements versus time data series. The corresponding corrected values of level measurements and time were used as data series to plot the trend line $L(t)$ using Microsoft Excel spreadsheet software, where the level drop rate, $\Delta L/t$, was obtained as its first order derivative, dL/dt. Actual level measurements are lower due to instrument error, e, taken at a reasonable value of e = 1% based on instrument catalogue and calibration test done.

The data series of level reading and time were charted using the Scatter chart, which is also known as the *XY* chart. The equation of the trend line is automatically shown as desired. In the Excel spreadsheet sample presented in Figure 2, the trend line is

$$y = -0.021x^2 + 4.786x + 192.9; \text{ where } y \text{ corresponds to level, } L; \text{ and, } x \text{ corresponds to time, } t.$$

Hence,

$$L = -0.021t^2 + 4.786t + 192.9$$

Therefore, the derivative of L with respect to t is:

$$dL/dt = -0.042t + 4.786$$

This derivative was used to calculate the succeeding values of level drop rates. In Figure 2, entry of these values for quick calculation of level drop rates at a particular time is shown in the formula bar of Microsoft Excel spreadsheet.

2.1.3 Analytical Calculation of Mean Coefficient of Discharge

The coefficient of discharge, C_d, can also be calculated analytically by measuring the time, t, required for the open tank to empty between initial level h_1 to final level, h_2, through the adapter as illustrated in Figure 3. However, the calculated value would be the average coefficient of discharge at these intervals.

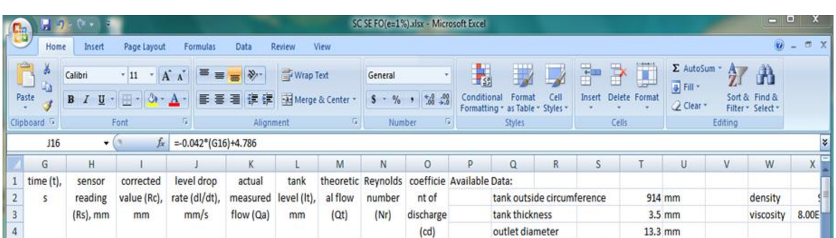


Figure 2. Formulation of level drop rates in Microsoft Excel

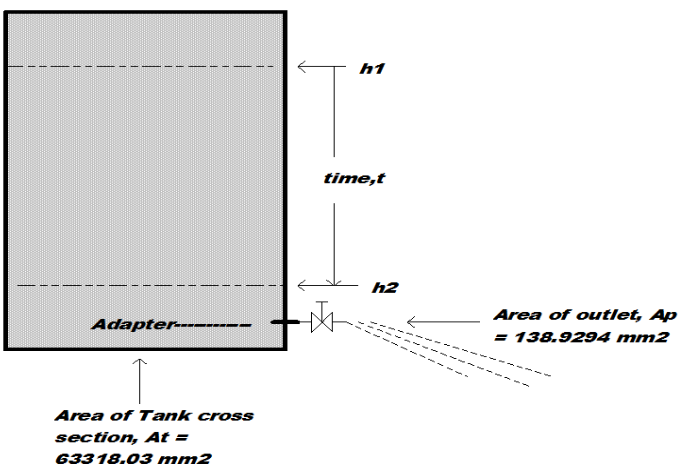


Figure 3. Open tank experiment model

From Torricelli's Law, the volume flow rate through the adapter, Q_t, is given by

$$Q_t = C_d A_p (2gh)^{1/2} \tag{7}$$

On the other hand, the volume removed from the tank, Q_a, is given by

$$Q_a = - A_t \; dh/dt; \tag{8}$$

with (-) sign indicating a decreasing *dh/dt.*

These two volumes (Q_a and Q_t) must be equal. Hence,

$$- A_t \; dh/dt = C_d A_p \, (2gh)^{1/2} \tag{9}$$

Solving for C_d from equation (9),

$$C_d = \frac{2\,(h_1^{\,1/2} - h_2^{\,1/2})}{(2g)^{1/2}\,(A_p / A_t)\,(t_2 - t_1)} \tag{10}$$

where

$t = (t_2 - t_1)$ is the time.

This value of C_d as calculated analytically can then be used to counter check the consistency of numerical calculations performed using Microsoft Excel spreadsheet by taking the average C_d of all the data points,

$$C_{d\ average} = \frac{\sum_1^n C_{d\ n}}{n} \tag{11}$$

Once C_d is known, the time for discharge can be solved using the following formula

$$t = \frac{2\,(h_1^{1/2} - h_2^{\,1/2})}{C_d\,(2g)^{1/2}\,(A_p / A_t)} \tag{12}$$

2.2 Experimental Method

The research procedure was performed in accordance with the laws governing open tank discharge applications, most specifically Torricelli's Law. The research involved set-up preparation, commissioning, calibration, data acquisition and data processing.

A standard 20 liter container was modified into a tank for conducting basic draining experiment. The tank was designed for incorporation of UPVC pipe adapters and an ultrasonic level sensor measuring device. Figure 4 shows the experimental setup. It is composed of two major parts; namely: the hydraulic system and the instrumentation system.

The hydraulic system involves the draining and supplying of water and vice versa. On the other hand, the instrumentation is responsible for the measurements and logging, automation and control of the entire system.

In the preparation of the adapters, special tools were used for cutting and edge tapering. A 20 mm NELTEX UPVC pipe male adapter was used in the study. Different geometry has been considered and their effects to the behavior of flow were identified. Figure 5 shows the different adapter geometry used in the investigation.

Before the experimental setup was put into usable condition, preliminary test were done to ensure the best performance and reliability of tests results. These tests are the ground leveling of the entire test equipment, hydraulic leak test of the draining tank model, and test run setup and data recording.

Calibration was done thereafter by comparing the ultrasonic level reading and the measurement reading using a standard vernier caliper meter with an accuracy of 0.05 mm.

Calibration of the open tank model discharge done using the conventional "bucket and stop watch method". Results are then tabulated in Table 1. Parameterization software ULTRA 2001 was used to configure the instrument for data logging.

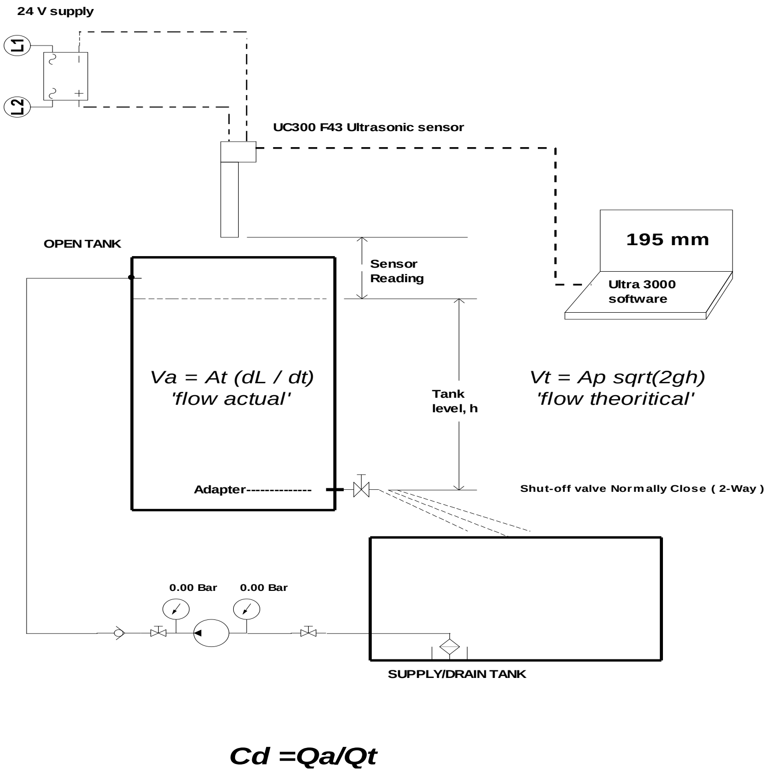
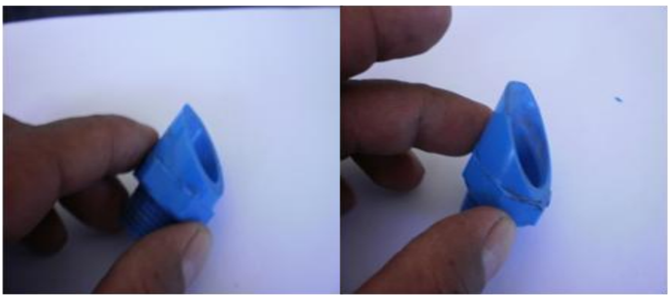


Figure 4. Experimental Setup

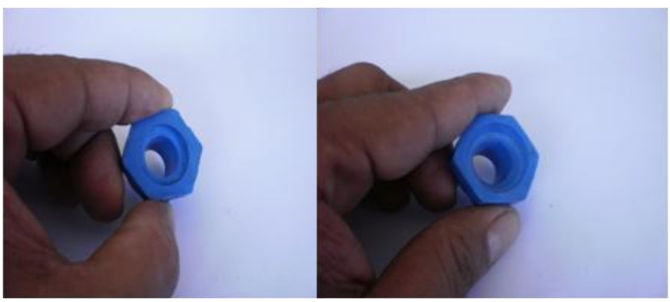
2.3 Method of Analysis

The logged data was automatically stored in the program file folder location. These logged data will then be saved as a text file (.txt) with their appropriate filenames. At least five (5) trials were logged for all adapter geometries. The logged data were then exported to Microsoft Excel spreadsheet for processing. The data series of the corrected level readings and their respective time were charted using the Excel spreadsheet scatter chart which is also known as the *XY* chart. The equation of the trend line was then automatically shown as desired. Using these equations, the first derivatives were determined which are equal to the instantaneous level drop rates, *dL/dt* at time *t*. The equations of the derivatives were then used to calculate all the remaining drop rates. Consequently, other necessary values were automatically solved by Microsoft Excel as formulated using initial data available.

a. (long, squared) b. (long, tapered) c. (long45, squared)



d. (long30, squared) e. (long45, tapered)



f. (short, squared) g. (short, tapered)

Figure 5. Different adapter geometry used in the study

Table 1. Experimental results using conventional bucket and stop watch method

Available data				
		Uncorrected		
Tank area	63,318.03mm^2	1	Sensor reading volume	6648.39315 mm^3
Test range	195 mm - 300 mm	2	Using ave. *dL/dt*	6639.19621mm^3
Discharge time	22.75 s	*Corrected*		
Uncorrected ave. *dL/dt*	4.609 mm/s	3	Sensor reading volume	6582.56748 mm^3
Corrected ave. *dL/dt* at e = 1%	4.558 mm/s	4	Using ave. *dL/dt*	6565.73146 mm^3

Volume discharge from 195 mm to 300 mm sensor reading		Percent (%) error relative to			
		Uncorrected		Corrected	
	(cm^3)	1	2	3	4
Trial 1	6602	-0.70	-0.56	0.29	0.55
Trial 2	6595	-0.81	-0.67	0.19	0.44
Trial 3	6568	-1.22	-1.08	-0.22	0.03
Trial 4	6604	-0.67	-0.53	0.32	0.58
Trial 5	6578	-1.07	-0.93	-0.07	0.19
Average	6589.4	-0.90	-0.76	0.1	0.36

3. Results and Discussion

In the preliminary results in Table 2, the initial level drop rates (in *mm/s*) at 195 mm starting level were determined for each adapter and instantly gave the first clear observation to which geometry have higher values of coefficient of discharge. The higher the level drop rates the lower resistance and pressure loss in the adapter. The shorter and tapered edged adapter had the highest level drop rates while the angle of cut is not a factor in increasing the level drop rates or flow rates.

Table 2. Level drop rates

Adapter geometry	Initial drop rate at initial level (*mm/s*)	Drop deceleration (*mm^2/s*)	Volume discharge from 195 mm in 65s	% volume difference from original adapter
short, tapered (SC TE)	5.12	0.048	14607.03	4.8
long, tapered (LC TE)	4.84	0.042	14193.27	1.8
short, squared (SC SE)	4.77	0.042	14030.27	0.6
long, squared (LC SE)	4.70	0.040	13942.5	0.0
long (30cut), squared (30LC SE)	4.69	0.040	13911.07	-0.2
long (45cut), tapered (45LC TE)	4.67	0.040	13854.74	-0.6
long (45cut), squared(45LC SE)	4.56	0.038	13704.28	-1.7

Figure 6 shows the relationship of coefficient of discharge, C_d, against head, h, and Reynolds number, N_R. It should be emphasized that the resulting values are consistent with Mott, Richard (1994) and ASME computed values for venturi and nozzles at 0.94– 0.98 range. In general, the values of C_d changes with h and N_R. C_d changes gradually at higher values, however, changes rapidly at lower values. This means that these meters perform better measurements if used at higher h and $N_{R.}$

The mean values of C_d can be determined numerically using Excel and analytically using Torricelli's Law as shown in Table 3. The mean value of coefficient of discharge is the typical value used for approximate calculations.

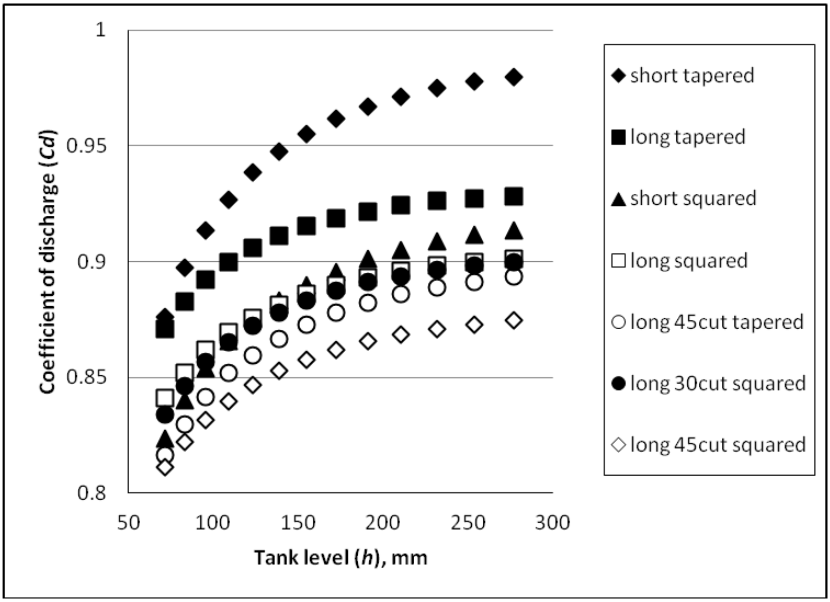
4. Conclusion and Recommendation

Initial calculations of values of level drop rates gave the initial impression to the performance of the meters. Higher drop rates indicated better performance since it translates to higher discharge rates and faster discharge times.

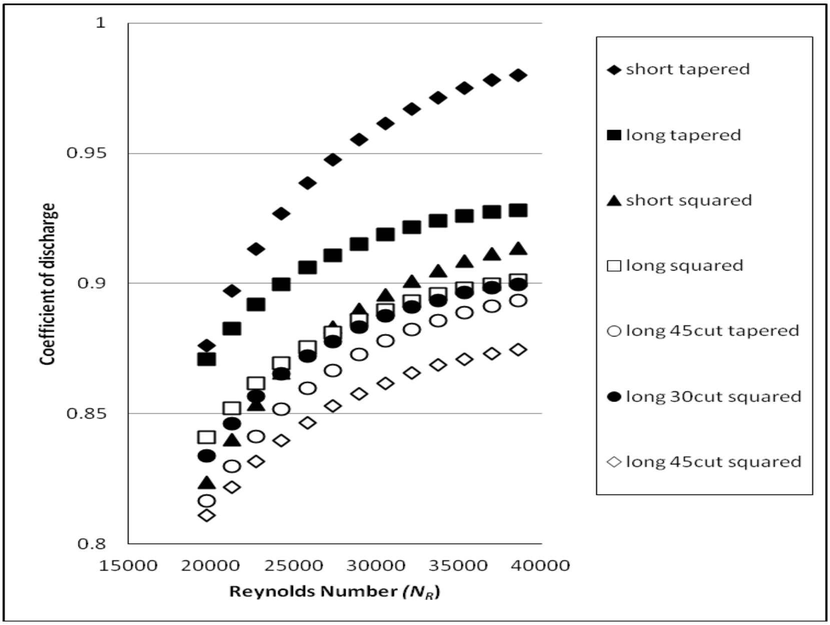
The shorter and tapered geometry produced higher values of coefficient of discharge with respect to the head and the Reynolds number; therefore it is the best configuration to use. The angled cut variation produced unfavorable results to the flow rates.

The analytical and numerical values of mean coefficient of discharge are consistent with those in related studies; therefore these mean values can be used for flow rate approximations.

The use of UPVC pipe male adapters as variable head flow meters is recommended as an alternative variable head flow meter for simple applications like open tank discharge metering. However, calibrations of the meter should be done beforehand using the procedure presented in this study. The same calibration data gathered is to be used respectively with the same meter.



(a) Coefficient of discharge, C_d vs. head, h



(b) Coefficient of discharge, C_d vs. Reynolds number, N_R

Figure 6. Relationships between coefficient of discharge, C_d against (a) head, h; and, (b) Reynolds number, N_R

Table 3. Mean values of C_d at $20000<N_R<40000$

Adapter geometry	SC TE	LC TE	SC SE	LC SE	45LC TE	30LC SE	45LC SE
Numerical value	0.9449	0.9093	0.8818	0.8802	0.8658	0.8769	0.8519
Analytical value	0.9414	0.9012	0.8827	0.8767	0.8665	0.8871	0.8523
% error	0.3719	0.8999	0.0993	0.3978	0.0721	1.1477	0.0496

UPVC pipe male adapters are generally manufactured having standard squared edge. In order to achieve higher coefficient of discharges; edge preparations like tapering or rounding of inlet edges and corners must be performed using simple machining tools like sandpaper or drill mounted aluminum oxide stone of varied shapes and angles.

Results showed that shorter and tapered geometry provides the highest coefficient of discharges. For this reason cutting is a recommended option, although there is a need for extra effort and time for cutting the standard-sized adapters. Care should be observed during cutting and installation because results also showed that angular variations produce negative effects to flow rates.

5. References

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